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Revisiting 802.11e EDCA performance analysis

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Abstract In this short communication we focus on the Contention Window mechanism of 802.11e EDCA. We first show that existing analyses for this mechanism rely on a system of equations that does not always have a unique solution. Next, we present an alternative analysis for which we prove mathematically the existence of a unique solution. Finally, we validate the accuracy of our analysis by means of simulation results.

Keywords WLAN · 802.11e · EDCA · CW mechanism · Performance analysis

1 Introduction

Existing performance analyses of the Contention Window (CW) mechanism of IEEE 802.11e EDCA [1,4–6] rely on a system of equations that does not always have a unique solution. This represents a major drawback as systems with multiple solutions are known to suffer from a number of problems. In this letter, we propose a novel model that, in contrast to the those analyses, always has a unique solution.

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The CW mechanism of EDCA is briefly summarized as follows. A station contending for transmission initializes its backoff time counter to a random value uniformly distributed in the range $(0, CW - 1)$, where CW is the station's CW and is initially set equal to a value CW_{min} (called minimum CW). The backoff time counter is decremented once every slot time as long as the channel is sensed idle, "frozen" when a transmission is detected, and reactivated when the channel is sensed idle again for a certain period.

Once the backoff counter reaches zero, the station transmits. A collision occurs when two or more stations transmit simultaneously. After an unsuccessful transmission, CW is doubled, up to a maximum value $CW_{max} = 2^m CW_{min}$ (where m is the station's maximum backoff stage) and the backoff process is restarted. Once the backoff process completes with a successful transmission, CW is set again equal to CW_{min}.

As it can be seen from the above description, the behavior of a station depends on two parameters, namely the minimum Contention Window (CW_{min}) and the maximum backoff stage (m). These are open configurable parameters that can be set to different values for different Access Categories (AC's). The rest of the paper is devoted to the analysis of the CW mechanism as a function of the number of stations and parameter configuration of each AC.

2 Problem of Bianchi's derived models

To the authors' knowledge, all previous analyses of EDCA (see, e.g., [1,4–6]) are implicitly or explicitly derived from Bianchi's model of 802.11 DCF [2] as follows. According to [2], the probability that a station of AC i transmits in a randomly chosen slot time is given by

$$\tau_i = \frac{2}{1 + W_i + c_i\, W_i \sum_{j=0}^{m_i-1} (2c_i)^j}, \tag{1}$$

where W_i and m_i are the minimum CW and maximum backoff stage of AC i, respectively, and c_i is the probability that the transmission of a station of AC i collides.

To compute c_i, [1,4–6] rely on the assumption of [2] that each station transmits with a constant and independent probability. This yields

$$c_i = 1 - (1-\tau_i)^{n_i-1} \prod_{j\neq i} (1-\tau_j)^{n_j}, \tag{2}$$

where n_i is the number of stations of AC i.

The above two expressions form a system of equations on the τ_i's. Indeed, each τ_i can be expressed as a function of all the τ_i's as follows: from (2), each c_i can be computed as a function of all τ_i's, and from (1) we can then obtain each τ_i from the corresponding c_i. This therefore comprises a system of N nonlinear equations on the τ_i's, where N is the number of AC's in the WLAN. The analyses of [1,4–6] propose to resolve this system by using numerical techniques.

A major problem with the above analysis is that the system of equations upon which it relies is not guaranteed to have a unique solution. This is proved by means of the following counterexample. Consider a WLAN with one station of AC 1 and another of AC 2, with $\{W_1 = 2, m_1 = 5\}$ and $\{W_2 = 2, m_2 = 6\}$. If we look at the $\{\tau_1, \tau_2\}$ values that satisfy the system of equations resulting from this configuration, we find as many as three solutions to the system, namely $\{0.237, 0.514\}$, $\{0.318, 0.431\}$, and $\{0.589, 0.142\}$. None of these solutions match the values that we obtain via simulation, which are $\{0.411, 0.318\}$.

The above example shows that it cannot be guaranteed that the system of equations of [1,4–6] has only one solution. This represents a major drawback, as numerical techniques may suffer from convergence problems with systems that have multiple solutions. In addition, even if the system converges to one of the solutions, this solution may not be the closest one to the real point of operation. In fact, in our example none of the solutions of the system are close to the operation point obtained from simulation.

In the next section, we present a novel model to analyze the CW mechanism of EDCA that, unlike the previous analyses of EDCA, is based on a system of equations which is guaranteed to have a unique solution.

3 Novel EDCA model

The initial assumption of our model is the one of [3] that backoff times follow a geometric distribution. With this assumption, a station of AC i transmits in a slot time with an independent probability equal to

$$\tau_{i,j} = \frac{2}{2^j W_i + 1}, \tag{3}$$

where j is the backoff stage of the station, i.e., the number of times that the CW of the station has been doubled since the last successful transmission.

Let us consider the simplified model in which one station of AC 1 and one station of AC i behave according to the given backoff rules, while the probability that some other station transmits in a slot time is given by a constant and independent probability equal to p_i. With the given assumptions, this system can be modeled as a bidimensional stochastic process $\{j, k\}$, where j is the backoff stage of the station of AC 1 and k the backoff stage of the station of AC i. This process is characterized by the Markov chain depicted in Fig. 1.

We next analyze the transition probabilities between the states of the Markov chain. A successful transmission occurs when one station transmits and none of the other stations transmit. This yields the following transition probabilities:

$$p_{\{j,k\}\to\{0,k\}} = \tau_{1,j}(1-\tau_{i,k})(1-p_i), \tag{4}$$

$$p_{\{j,k\}\to\{j,0\}} = \tau_{i,k}(1-\tau_{1,j})(1-p_i). \tag{5}$$

When one of the two considered stations transmits (either the one of AC 1 or the one of AC i),

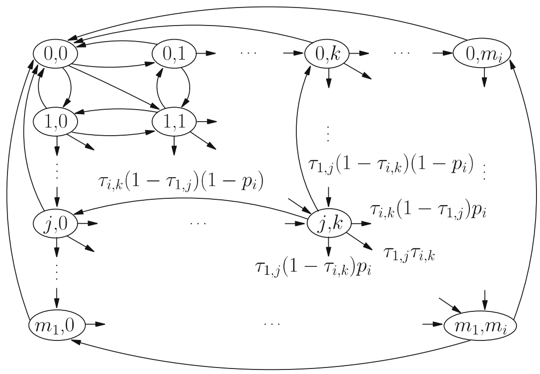

Fig. 1 Markov chain of process $\{j, k\}$

the other does not but some other station transmits, this leads to increasing the backoff stage of the transmitting station:

$$p_{\{j,k\}\to\{j+1,k\}} = \tau_{1,j}(1-\tau_{i,k})p_i, \tag{6}$$

$$p_{\{j,k\}\to\{j,k+1\}} = \tau_{i,k}(1-\tau_{1,j})p_i. \tag{7}$$

Finally, if the two considered stations transmit, they both increase their backoff stage:

$$p_{\{j,k\}\to\{j+1,k+1\}} = \tau_{1,j}\tau_{i,k}. \tag{8}$$

The above Markov chain can be resolved using standard techniques and the state probabilities $P_{j,k}$ can thus be obtained. Given the $P_{j,k}$'s, the transmission probabilities of the two considered stations can be computed as follows:

$$\tau_1 = \sum_{j=0}^{m_1}\sum_{k=0}^{m_i} P_{j,k}\tau_{1,j}, \tag{9}$$

$$\tau_i = \sum_{j=0}^{m_1}\sum_{k=0}^{m_i} P_{j,k}\tau_{i,k}. \tag{10}$$

From the above, we have that, given the probability p_i that some station other than the two considered ones transmits, we can resolve the Markov chain and, from the Markov chain state probabilities, we can compute the transmission probabilities of the two considered stations. Hereafter, we refer with $\tau_1(p_i)$ and $\tau_i(p_i)$ to the transmission probabilities of the two stations of AC 1 and AC i, respectively, computed as a function of p_i.

Following the above, by considering the simplified models corresponding to different i values, we can compute τ_1 and τ_i as a function of p_i for all i in $\{2, \ldots, N\}$. This makes our analysis dependent on $N-1$ variables, namely p_i for $i \in \{2, \ldots, N\}$. To resolve these $N-1$ variables, we need to build a system of $N-1$ equations. The first $N-2$ equations of our system are the ones resulting from forcing that τ_1 always takes the same value independent of the AC i chosen for the simplified model:

$$\tau_1(p_i) = \tau_1(p_{i+1}),\ i \in \{2, \ldots, N-1\}. \tag{11}$$

With the assumption that all the stations other than the two considered ones transmit with a constant and independent probability, p_i can be computed as:

$$p_i = 1 - (1-\tau_1)^{n_1-1}(1-\tau_i)^{n_i-1} \times \prod_{k\neq 1,i}(1-\tau_k)^{n_k}. \tag{12}$$

Since the τ_i's can be expressed as a function of the p_i's, we have that the above expression represents an equation on the p_i's. The last equation of our system is the result from considering the above expression for all i in $\{2, \ldots, N\}$ and multiplying all the respective left and right hand sides:

$$\prod_{i=2}^{N} p_i = \prod_{i=2}^{N}\left(1-(1-\tau_1(p_i))^{n_1-1}(1-\tau_i(p_i))^{n_i-1} \times \prod_{k\neq 1,i}(1-\tau_k(p_k))^{n_k}\right). \tag{13}$$

With the above, we have a system of $N-1$ nonlinear equations on the p_i's, which terminates the analysis. The computational complexity of this analysis is similar to the one of the models presented in Sect. 2, as those models require solving a system of N equations and ours a system of $N-1$ equations.

The main strength of the presented model as compared to the analyses of Sect. 2 is that our model, in contrast to those analyses, relies on a system of equations that is *guaranteed to have a unique solution*. This is proved by Theorem 1 in the Appendix. Indeed, if we look at the example presented in Sect. 2, it can be seen that for the considered configuration there only exists one $\{\tau_1, \tau_2\}$ pair of values that satisfies the above system of equations. The solution, $\{0.416, 0.324\}$, closely matches the simulation results reported in Sect. 2.

4 Throughput evaluation

The model presented in the previous section gives the transmission probabilities τ_i of all the AC's. Based on this, the throughput of all the AC's in the WLAN can be computed as follows. According to [2], the throughput experienced by a station of AC i is given by

$$r_i = \frac{p(s_i)l}{p(s)T_s + p(c)T_c + p(e)\sigma}, \tag{14}$$

where $p(s_i)$ is the probability that a randomly chosen slot time contains a successful transmission of a given station of AC i, l is the average payload size of a transmission, $p(s)$, $p(c)$, and $p(e)$ are the probabilities that a slot time contains a successful transmission, a collision or is empty, respectively, and T_s, T_c, and σ are the average slot time durations in each case. The probabilities $p(e), p(s), p(c)$, and $p(s_i)$ are calculated as follows:

$$p(e) = \prod\nolimits_i (1-\tau_i)^{n_i}, \tag{15}$$

$$p(s_i) = \tau_i(1-\tau_i)^{n_i-1} \prod\nolimits_{j\neq i} (1-\tau_j)^{n_j}, \tag{16}$$

$$p(s) = \sum\nolimits_i n_i p(s_i), \tag{17}$$

$$p(c) = 1 - p(e) - p(s). \tag{18}$$

In order to validate the accuracy of our model, we compared the throughput results obtained analytically against those obtained via simulation. The simulations were performed for a WLAN with the system parameters of the IEEE 802.11b physical layer. We considered four AC's (AC $i \in \{1, \ldots, 4\}$) configured as follows. The minimum Contention Window of AC 1, W_1, was set to different values for different simulations, and the minimum Contention Window of the other AC's was set equal to $W_1 2^{i-1}$. The maximum backoff stage was set equal to 5 for all AC's. There were five stations of each AC and the packet size was set equal to 1,500 bytes.

Figure 2 plots the throughput values obtained analytically (lines) and via simulation (points) from the above scenario. Simulation results are plotted with 95% confidence interval bars (although these are so small that they can barely be appreciated in the graph). We observe from the figure that analytical results match simulations very closely, which confirms the accuracy of our model.

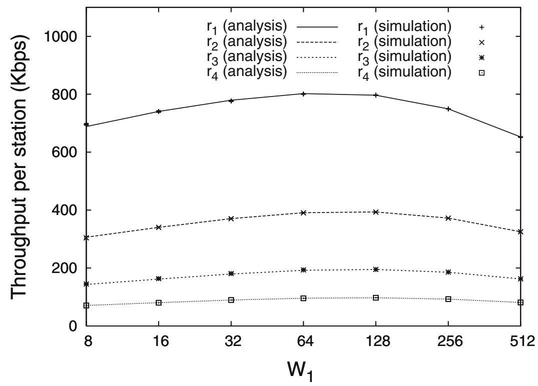

Fig. 2 Throughput: analytical and simulation results

5 Final remarks

In this letter, we have presented a novel model to analyze the CW mechanism of 802.11e EDCA. The main merits of our model are: (1) the analysis relies on a system of nonlinear equations that is guaranteed to have a unique solution, which avoids the convergence and accuracy problems that previous analyses suffer from, (2) computational efficiency is similar to previous models, as the size of the system of equations to be solved is similar for both cases, and (3) the model is very accurate, as we have validated via simulation.

Appendix

Theorem 1 *There exists one and only one solution to the system of $N-1$ nonlinear equations formed by (11) and (13).*

Proof We first show that, starting from a given p_i, we can express all the other p_i's as an increasing function of this p_i that grows up to 1. We start by noting that from the resolution of the Markov chain of (4)–(8) it can be seen that for all i, $\tau_1(p_i)$ and $\tau_i(p_i)$ are decreasing functions of p_i, and for all i and j, $\tau_1(p_i)|_{p_i=1} = \tau_1(p_j)|_{p_j=1}$. Next, we consider the AC k for which $\tau_1(p_k)|_{p_k=0}$ is minimum and show that all p_i's can be computed as a function of p_k. This follows from (11). Indeed, $\tau_1(p_k)$ is fixed for a given p_k, while $\tau_1(p_i)$ is a decreasing function on $p_i \in (0, 1)$ which, as it can be seen from the above properties of $\tau_1(p_i)$, starts from a value that (with our choice of k) is not smaller than $\tau_1(p_k)$

and decreases down to a value that is not larger than $\tau_1(p_k)$. Therefore, it follows that there exists only one value of p_i that satisfies $\tau_1(p_i) = \tau_1(p_k)$. In addition, from the fact that $\tau_1(p_i)$ and $\tau_1(p_k)$ are both decreasing functions, we have that this value of p_i increases with p_k, and from $\tau_1(p_i)|_{p_i=1} = \tau_1(p_k)|_{p_k=1}$, we have that it reaches 1 when $p_k = 1$.

Let us now focus on (13). From the above, we have that the left hand side of (13) can be expressed as an increasing function of p_k that starts from 0 and grows up to 1 in $p_k \in (0, 1)$. Since (as shown above) τ_i is a decreasing function of p_i and p_i an increasing function of p_k, we have that the right hand side of (13) can be expressed as a decreasing function of p_k. Furthermore, it can be seen that this function starts from a value not smaller than 0 and reduces down to a value not larger than 1. From this, it follows that there exists only one value of p_k that satisfies condition (13). Taking the resulting value of p_k and undoing all the previous steps, we have a solution to the system. Uniqueness of the solution is given by the fact that all relationships are bijective and any solution must satisfy (13) which, as we have shown, has only one solution. □

References

1. Banchs, A., & Vollero, L. (2005). A Delay model for 802.11e EDCA. *IEEE Communications Letters, 9*(6).
2. Bianchi, G. (2000). Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE Journal on Selected Areas in Communications, 18*(3).
3. Cali, F., Conti, M., & Gregori, E. (2000). Dynamic tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit. *IEEE/ACM Transactions on Networking, 8*(6).
4. Hui, J., & Devetsikiotis, M. (2005). A unified model for the performance analysis of IEEE 802.11e EDCA. *IEEE Transactions on Communications, 53*(9).
5. Kong, Z., Tsang, D. H. K., Bensaou, B., & Gao, D. (2004). Performance analysis of IEEE 802.11e contention-based channel access. *IEEE Journal on Selected Areas in Communications, 22*(10).
6. Robinson, J. W., & Randhawa, T. S. (2004). Saturation throughput analysis of IEEE 802.11e enhanced distributed coordination function. *IEEE Journal on Selected Areas in Communications, 22*(5), 917–928.

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